CIERTAS RELACIONES DE RECURRENCIA PARA LA FUNCION KAMPE DE FERIET

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1. La función de Fériet, con una ligera modificación de notación, será representada y definida del modo siguiente:

$$(1.1)\qquad F^{\lambda,\,\mu}_{\nu,\,\sigma}\begin{pmatrix}\alpha_\lambda:\ \beta_\mu,\ \beta'_\mu\\ \gamma_\nu:\ \rho_\sigma,\ \rho'_\sigma\end{pmatrix}=\sum_{m=0}^{\infty}\ \sum_{n=0}^{\infty}\frac{(\alpha_\lambda,\ m+n)\,(\beta_\mu,\ m)\ (\beta'_\mu,\ n)\,x^m y^n}{m!\,n!\,(\gamma_\nu,\ m+n)\,(\rho_\sigma,\ m)\,(\rho'_\sigma,\ n)}$$

donde $(\alpha, m)=\alpha(\alpha+1)\ \ldots\ (\alpha+m-1)$; $(\alpha, 0)=1$ y α_λ denotan la secuencia de parámetros $\alpha_1,\ \alpha_2,\ \ldots\ \alpha_\lambda$. Es absolutamente convergente cuando $\lambda+\mu\leq\nu+\sigma+1$.

2. Resultados que se usarán en este trabajo [1, 2]

$$(2.1)\qquad \int_0^\infty x^{hs-1}\,G^{m,\,n}_{p,\,q}\left(ax^h\,\middle|\,\begin{matrix}a_1,\ a_2,\ \ldots,\ a_p\\ b_1,\ b_2,\ \ldots,\ b_q\end{matrix}\right)F^{\lambda,\,\mu}_{\nu,\,\sigma}\left(\begin{matrix}\alpha_\lambda:\ \beta_\mu,\ \beta'_\mu\\ \gamma_\nu:\ \rho_\sigma,\ \rho'\end{matrix}\,\middle|\,cx^{th},\ dx^{th}\right)dx$$

$$=\frac{1}{ha^s}\,\frac{\prod\limits_{i=1}^{m}\Gamma(b_i+s)\ \prod\limits_{i=1}^{n}(1-a_i-s)}{\prod\limits_{i=m+1}^{q}\Gamma(1-b_i-s)\ \prod\limits_{i=n+1}^{p}(a_i+s)}\,F^{\lambda+tq,\,\mu}_{\nu+tp,\,\sigma}\left(\begin{matrix}\alpha_\lambda,\ \alpha^*:\ \beta_\mu,\ \beta'_\mu\\ \gamma_\nu,\ \gamma^*:\ \rho_\sigma,\ \rho'_\sigma\end{matrix}\,\middle|\,\frac{ct^{t(q-p)}}{a^t},\ \frac{dt^{t(q-p)}}{a^t}\right)$$

$$\text{donde } \alpha^*=\frac{b_i+s+j-1}{t}\ (i=1,\ 2,\ \ldots,\ q \text{ y } j=1,\ 2,\ \ldots,\ t)$$

$$\text{y}\quad \gamma^*=\frac{a_i+s+j-1}{t}\ (i=1,\ 2,\ \ldots,\ p \text{ y } j=1,\ 2,\ \ldots,\ t)$$

siempre que $p+q<2(m+n)$; $|\arg a|<(m+n-\tfrac{1}{2}p-\tfrac{1}{2}q)\pi$; $-\min_{1\leq j\leq m} R(b_j)<R(s)<1-\max_{1\leq j\leq n} R(a_j)$; $\lambda+\mu\leq\nu+\sigma+1$ y t sea un entero positivo. Cuando t es un entero negativo, α^* y γ^* se deben reemplazar por

$$\frac{1-a_i-s+j-1}{-t}\;(i=1, 2, \ldots, p;\;\; j=1, 2, \ldots, t)$$

$$\frac{1-b_i-s+j-1}{-t}\;(i=1, 2, \ldots, q;\;\; j=1, 2, \ldots, t)$$

respectivamente y el argumento por

$$\frac{c(-t)^{t(q-p)}}{a^t},\;\; \frac{d(-t)^{t(q-p)}}{a^t}.$$

(2.2
$$b_1-b_2)\,G^{m,\,n}_{p,\,q}\left(x\,\middle|\,\begin{matrix}a_1-1, a_2, \ldots, a_p\\ b_1-1, b_2-1, b_3, \ldots, b_q\end{matrix}\right)$$
$$=(a_1-b_1)\,G^{m,\,n}_{p,\,q}\left(x\,\middle|\,\begin{matrix}a_1, \ldots, a_p\\ b_1-1, b_2, \ldots, b_q\end{matrix}\right)$$
$$-(a_1-b_2)\,G^{m,\,n}_{p,\,q}\left(x\,\middle|\,\begin{matrix}a_1, \ldots, a_p\\ b_1, b_2-1, b_3, \ldots, b_q\end{matrix}\right)$$

donde $m\geq 2$ y $n\geq 1$.

3. Resultados por demostrar:

(3.1)
$$(b_1-b_2)\,(1-a_1-s)\,F^{\lambda+q,\,\mu}_{\nu+p,\,\sigma}\left(\begin{matrix}\alpha_\lambda, b_1+s-1, b_2+s-1, \ldots,\\ \gamma_\nu, a_1+s-1, a_2+s, \ldots,\end{matrix}\right.$$
$$\left.\begin{matrix}b_q+s : \beta_\mu, \beta'_\mu\\ a_p+s : \rho_\sigma, \rho'_\sigma\end{matrix}\,\middle|\, c, d\right)$$
$$=(a_1-b_1)\,(b_2+s-1)\,F^{\lambda+q,\,\mu}_{\nu+p,\,\sigma}\left(\begin{matrix}\alpha_\lambda, b_1+s-1, b_2+s, \ldots,\\ \gamma_\nu, a_1+s, a_2+s, \ldots,\end{matrix}\right.$$
$$\left.\begin{matrix}b_q+s : \beta_\mu, \beta'_\mu\\ a_p+s : \rho_\sigma, \rho'_\sigma\end{matrix}\,\middle|\, c, d\right)$$

$$-(a_1-b_2)(b_1+s-1)F^{\lambda+q,\ \mu}_{\nu+p,\ \sigma}\left(\begin{matrix}\alpha_\lambda,\ b_1+s,\ b_2+s-1,\ b_3+s,\ \ldots,\\ \gamma_\nu,\ a_1+s,\ a_2+s,\ a_3+s,\ \ldots,\end{matrix}\right.$$

$$\left.\begin{matrix}b_q+s:\beta_\mu,\ \beta'_\mu\\ a_p+s:\rho_\sigma,\ \rho'_\sigma\end{matrix}\ \middle|\ c,\ d\right).$$

(3.2)
$$\frac{(a_p-1)(b_1+s-1)(b_2+s-1)}{(1-a_1-s)(a_p+s-1)}F^{\lambda+q,\ \mu}_{\nu+p,\ \sigma}\left(\begin{matrix}\alpha_\lambda,\ b_1+s,\ \ldots,\\ \gamma_\nu,\ a_1+s,\ \ldots,\end{matrix}\right.$$

$$\left.\begin{matrix}b_q+s:\beta_\mu,\ \beta'_\mu\\ a_p+s:\rho_\sigma,\ \rho'_\sigma\end{matrix}\ \middle|\ c,\ d\right)$$

$$=\frac{(b_1+s-1)(b_2+s-1)}{(1-a_1-s)}F^{\lambda+q,\ \mu}_{\nu+p,\ \sigma}\left(\begin{matrix}\alpha_\lambda,\ b_1+s,\ \ldots,\\ \gamma_\nu,\ a_1+s,\ \ldots,\end{matrix}\right.$$

$$\left.\begin{matrix}b_q+s:\beta_\mu,\ \beta'_\mu\\ a_p+s-1:\rho_\sigma,\ \rho'_\sigma\end{matrix}\ \middle|\ c,\ d\right)$$

$$+\frac{(a_p-b_1)(a_p-b_2)}{(a_p+s-1)}F^{\lambda-q,\ \mu}_{\nu+p,\ \sigma}\left(\begin{matrix}\alpha_\lambda,\ b_1+s-1,\ b_2+s-1,\ \ldots,\\ \gamma_\nu,\ a_1+s-1,\ a_2+s,\ \ldots,\end{matrix}\right.$$

$$\left.\begin{matrix}b_q+s:\beta_\mu,\ \beta'_\mu\\ a_p+s:\rho_\sigma,\ \rho'_\sigma\end{matrix}\ \middle|\ c,\ d\right)$$

$$+(1+b_1+b_2-2a_p)F^{\lambda+q,\ \mu}_{\nu-p,\ \sigma}\left(\begin{matrix}\alpha_\lambda,\ b_1+s-1,\ b_2+s-1,\ b_3+s,\ \ldots,\\ \gamma_\nu,\ a_1+s-1,\ a_2+s,\ \ldots,\end{matrix}\right.$$

$$\left.\begin{matrix}b_q+s:\beta_\mu,\ \beta'_\mu\\ a_{p-1}+s,\ a_p+s-1:\rho_\sigma,\ \rho'_\sigma\end{matrix}\ \middle|\ c,\ d\right)$$

$$+(a_p+s-2)(F^{\lambda+q,\ \mu}_{\nu-p,\ \sigma}\left(\begin{matrix}\alpha_\lambda\cdot\cdot,\ b_1+s-1,\ b_2+s-1,\ b_3+s,\ \ldots,\\ \gamma_\nu,\ a_1+s-1,\ a_2+s,\ \ldots,\end{matrix}\right.$$

$$\left.\begin{matrix}b_q+s:\beta_\mu,\ \beta'_\mu\\ a_p+s,\ a_p+s-2:\rho_\sigma,\ \rho'_\sigma\end{matrix}\ \middle|\ c,\ d\right).$$

(3.3)
$$\frac{(b_1-a_1-1)(b_l+s-1)(b_2+s-1)}{(1-a_1-s)(a_p+s-1)}F^{\lambda+q,\ \mu}_{\nu+p,\ \sigma}\left(\begin{matrix}\alpha_\lambda,\ b_1+s,\ \ldots,\\ \gamma_\nu,\ a_1+s,\ \ldots,\end{matrix}\right.$$

$$\begin{matrix} b_q+s: \beta_\mu, \beta'_\mu \\ a_p+s: \rho_\sigma, \rho'_\sigma \end{matrix} \left| c, d \right)$$

$$=\frac{(b_1+s-1)(b_2+s-1)(b_1+s)}{(1-a_1-s)(a_p+s-1)} F^{\lambda+q,\,\mu}_{\nu+p,\,\sigma}\left(\begin{matrix} \alpha_\lambda, b_1+s-1, b_2+s, \ldots, \\ \gamma_\nu, a_1+s, a_2+s, \ldots, \end{matrix}\right.$$

$$\begin{matrix} b_q+s: \beta_\mu, \beta'_\mu \\ a_p+s: \rho_\sigma, \rho'_\sigma \end{matrix} \left| c, d \right)$$

$$+(b_1+s-1) F^{\lambda+q,\,\mu}_{\nu+p,\,\sigma}\left(\begin{matrix} \alpha_\lambda, b_1+s, b_2+s-1, b_3+s, \ldots, \\ \gamma_\nu, a_1+s-1, a_2+s, a_3+s, \ldots, \end{matrix}\right.$$

$$\begin{matrix} b_q+s: \beta_\mu, \beta'_\mu \\ a_{p-1}+s, a_p+s-1: \rho_\sigma, \rho'_\sigma \end{matrix} \left| c, d \right)$$

$$+\frac{(a_p-b_1)(a_p-b_2)(a_p-1)}{(a_p+s-1)} F^{\lambda+q,\,\mu}_{\nu+p,\,\sigma}\left(\begin{matrix} \alpha_\lambda, b_1+s-1, b_2+s-1, b_3+s, \ldots, \\ \gamma_\nu, a_1+s-1, a_2+s, a_3+s, \ldots, \end{matrix}\right.$$

$$\begin{matrix} b_q+s: \beta_\mu, \beta'_\mu \\ a_p+s: \rho_\sigma, \rho'_\sigma \end{matrix} \left| c, d \right)$$

$$+(b_2-a_p) F^{\lambda+q,\,\mu}_{\nu+p,\,\sigma}\left(\begin{matrix} \alpha_\lambda, b_1+s-1, b_2+s-1, b_3+s, \ldots, \\ \gamma_\nu, a_1+s-1, a_2+s, \ldots, \end{matrix}\right.$$

$$\begin{matrix} b_q+s: \beta_\mu, \beta'_\mu \\ a_p+s-1: \rho_\sigma, \rho'_\sigma \end{matrix} \left| c, d \right).$$

(3.4)
$$\frac{(b_1-a_1)(b_1+s-1)(b_2+s-1)}{(1-a_1-s)(a_p+s-1)} F^{\lambda+q,\,\mu}_{\nu+p,\,\sigma}\left(\begin{matrix} \alpha_\lambda, b_1+s, \ldots, \\ \gamma_\nu, a_1+s, \ldots, \end{matrix}\right.$$

$$\begin{matrix} b_q+s: \beta_\mu, \beta'_\mu \\ a_p+s: \rho_\sigma, \rho'_\sigma \end{matrix} \left| c, d \right)$$

$$=\frac{(b_1+s-1)(b_2+s-1)}{(1-a_1-s)} F^{\lambda+q,\,\mu}_{\nu+p,\,\sigma}\left(\begin{matrix} \alpha_\lambda, b_1+s, \ldots, \\ \gamma_\nu, a_1+s, \ldots, \end{matrix}\right.$$

$$\begin{matrix} b_q+s: \beta_\mu, \beta'_\mu \\ a_p+s: \rho_\sigma, \rho'_\sigma \end{matrix} \left| c, d \right)$$

$$+\frac{(b_1-a_p)(b_1-a_1)(b_2+s-1)}{(1-a_1-s)(a_p+s-1)} F^{\lambda+q,\,\mu}_{\nu+p,\,\sigma}\left(\begin{matrix} \alpha_\lambda, b_1+s-1, b_2+s, \ldots, \\ \gamma_\nu, a_1+s, a_2+s, \ldots, \end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s:\beta_\mu,\beta'_\mu \\ a_p+s:\rho_\sigma,\rho'_\sigma \end{matrix}\right| c,d\Bigg)$$

$$+(1+b_2-a_p)F^{\lambda+q,\,\mu}_{\nu+p,\,\sigma}\left(\begin{matrix}\alpha_\lambda,\,b_1+s-1,\,b_2+s-1,\,b_3+s,\,\ldots,\\ \gamma_\nu,\,a_1+s-1,\,a_2+s,\,a_3+s,\,:::,\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s:\beta_\mu,\beta'_\mu \\ a_p+s-1:\rho_\sigma,\rho'_\sigma \end{matrix}\right| c,d\Bigg)$$

$$+(a_p+s-2)F^{\lambda+q,\,\mu}_{\nu+p,\,\sigma}\left(\begin{matrix}\alpha_\lambda,\,b_1+s-1,\,b_2+s-1,\,\ldots,\\ \gamma_\nu,\,a_1+s-1,\,a_2+s,\,\ldots,\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s:\beta_\mu,\beta'_\mu \\ a_p+s-1:\rho_\sigma,\rho'_\sigma \end{matrix}\right| c,d\Bigg):$$

(3.5)
$$\frac{(1+b_1+b_2-a_1-a_p)(b_2+s-1)}{(1-a_1-s)(a_p+s-1)}F^{\lambda+q,\,\mu}_{\nu+p,\,\sigma}\left(\begin{matrix}\alpha_\lambda,\,b_1+s,\,\ldots,\\ \gamma_\nu,\,a_1+s,\,\ldots,\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s:\beta_\mu,\beta'_\mu \\ a_p+s:\rho_\sigma,\rho'_\sigma \end{matrix}\right| c,d\Bigg)$$

$$=\frac{(b_2-a_p)(b_2-a_1)}{(1-a_1-s)(a_p+s-1)}F^{\lambda+q,\,\mu}_{\nu+p,\,\sigma}\left(\begin{matrix}\alpha_\lambda,\,b_1+s,\,b_2+s-1,\,b_3+s,\,\ldots,\\ \gamma_\nu,\,a_1+s,\,a_2+s,\,a_3+s,\,\ldots,\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s:\beta_\mu,\beta'_\mu \\ a_p+s:\rho_\sigma,\rho'_\sigma \end{matrix}\right| c,d\Bigg)$$

$$+\frac{(b_1+s+1)(b_2+s-1)}{(1-a_1-s)(a_p+s-1)}F^{\lambda+q,\,\mu}_{\nu+p,\,\sigma}\left(\begin{matrix}\alpha_\lambda,\,b_1+s-1+b_2+s,\,\ldots,\\ \gamma_\nu,\,a_1+s,\,a_2+s,\,\ldots,\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s:\beta_\mu,\beta'_\mu \\ a_p+s:\rho_\sigma,\rho'_\sigma \end{matrix}\right| c,d\Bigg)$$

$$+F^{\lambda+q,\,\mu}_{\nu+p,\,\sigma}\left(\begin{matrix}\alpha_\lambda,\,b_1+s,\,b_2+s-1,\,b_3+s,\,\ldots,\\ \gamma_\nu,\,a_1+s-1,\,a_2+s,\,a_3+s,\,\ldots,\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s:\beta_\mu,\beta'_\mu \\ a_p+s-1:\rho_\sigma,\rho'_\sigma \end{matrix}\right| c,d\Bigg).$$

Demostración: Para (2.2) tenemos

$$(3.6)\qquad (b_1-b_2)\int_0^\infty x^{s-1}f(x)\,G^{m,n}_{p,q}\left(x\,\middle|\,\begin{matrix}a_1-1, a_2, \ldots, a_p\\ b_1-1, b_2-1, b_3, \ldots, b_q\end{matrix}\right)dx$$

$$=(a_1-b_1)\int_0^\infty x^{s-1}f(x)\,G^{m,n}_{p,q}\left(x\,\middle|\,\begin{matrix}a_1, \ldots, a_p\\ b_1-1, b_2, \ldots, b_q\end{matrix}\right)dx$$

$$-(a_1-b_2)\int_0^\infty x^{s-1}f(x)\,G^{m,n}_{p,q}\left(x\,\middle|\,\begin{matrix}a_1, \ldots, a_p\\ b_1, b_2-1, b_3, \ldots, b_q\end{matrix}\right)dx$$

siempre que existan las integrales indicadas. Ahora, si tomamos

$$f(x)=F^{\lambda+q,\,\mu}_{\nu,\,\sigma}\left(\begin{matrix}\alpha_\lambda : \beta_\mu, \beta'_\mu\\ \gamma_\nu : \rho_\sigma, \rho'_\sigma\end{matrix}\,\middle|\, cx, dx\right)$$

en (3.6) y evaluamos las integrales indicadas allí, con ayuda de (2.1), para obtener la relación de recurrencia (3.1), después de ligeros cambios en los parámetros.

Las restantes relaciones de recurrencia se pueden demostrar de modo similar, si se comienza con [Jain (1967), ecs. 3.2, 3.3, 3.4, 3.5] respectivamente en lugar de (2.2).

4. *Casos particulares*

Las siguientes relaciones de recurrencia para la función hipergeométrica se obtiene si se toma $\mu=0$, $\sigma=1$, $c=d=x$ en las relaciones de recurrencia (3.1) a (3.5) respectivamente.

$$(4.1)\qquad (b_1-b_2)\,(1-a_1-s)\;{}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda, b_1+s-1\;\; b_2+s-1, \ldots\\ \gamma_\nu, a_1+s-1, a_2+s, \ldots\end{matrix}\right.$$

$$\left.\begin{matrix}b_q+s, \tfrac{1}{2}(\rho_1+\rho_2), \tfrac{1}{2}(\rho+\rho_2-1)\\ a_p+s, \rho_1, \rho_2, \rho_1+\rho_2-1\end{matrix}\,\middle|\, x\right)$$

$$=(a_1-b_1)\,(b_2+s-1)\;{}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda, b_1+s-1, b_2+s, \ldots\\ \gamma_\nu, a_1+s, a_2+s, \ldots\end{matrix}\right.$$

$$\left.\begin{matrix}b_q+s, \tfrac{1}{2}(\rho_1+\rho_2), \tfrac{1}{2}(\rho_1+\rho_2-1)\\ a_p+s, \rho_1, \rho_2, \rho_1+\rho_2-1\end{matrix}\,\middle|\, x\right)$$

$$-(a_1-b_2)(b_1+s-1)\,{}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda, b_1+s, b_2+s-1, b_3+s, \ldots\\ \gamma_\nu, a_1+s, a_2+s, \ldots\end{matrix}\right.$$

$$\left.\begin{matrix}b_q+s, \tfrac{1}{2}(\rho_1+\rho_2), \tfrac{1}{2}(\rho_1+\rho_2-1)\\ a_p+s, \rho_1, \rho_2, \rho_1+\rho_2-1\end{matrix}\;\middle|\; x\right),$$

$\lambda+y\leq\nu+p+2.$

$$(4.2)\qquad \frac{(a_p-a_1)(b_1+s-1)(b_2+s-1)}{(1-a_1-s)(a_p+s-1)}\,{}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda, b_1+s, \ldots\\ \gamma_\nu, a_1+s, \ldots\end{matrix}\right.$$

$$\left.\begin{matrix}a_p+s, \rho_1, \rho_2, \rho_1+\rho_2-1\\ b_q+s, \tfrac{1}{2}(\rho_1+\rho_2), \tfrac{1}{2}(\rho_1+\rho_2-1)\end{matrix}\;\middle|\; x\right)$$

$$=\frac{(b_1+s-1)(b_2+s-1)}{(1-a_1-s)}\,{}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda, b_1+s, \ldots\\ \gamma_\nu, a_1+s, \ldots\end{matrix}\right.$$

$$\left.\begin{matrix}b_q+s, \tfrac{1}{2}(\rho_1+\rho_2), \tfrac{1}{2}(\rho_1+\rho_2-1)\\ a_p+s-1, \rho_1, \rho_2, \rho_1+\rho_2-1\end{matrix}\;\middle|\; x\right)$$

$$+\frac{(a_p-b_1)(a_p-b_2)}{(a_p+s-1)}\,{}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda, b_1+s-1, b_2+s-1, \ldots\\ \gamma_\nu, a_1+s-1, a_2+s, \ldots\end{matrix}\right.$$

$$\left.\begin{matrix}b_q+s, \tfrac{1}{2}(\rho_1+\rho_2), \tfrac{1}{2}(\rho_1+\rho_2-1)\\ a_p+s, \rho_1, \rho_2, \rho_1+\rho_2-1\end{matrix}\;\middle|\; x\right)$$

$$+(1+b_1+b_2-2a_p)\,{}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda, b_1+s-1, b_2+s-1, \ldots\\ \gamma_\nu, a_1+s-1, a_2+s, \ldots\end{matrix}\right.$$

$$\left.\begin{matrix}b_q+s, \tfrac{1}{2}(\rho_1+\rho_2), \tfrac{1}{2}(\rho_1+\rho_2-1)\\ a_p+s-1, \rho_1, \rho_2, \rho_1+\rho_2-1\end{matrix}\;\middle|\; x\right)$$

$$+(a_p+s-2)\,{}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda, b_1+s-1, b_2+s-1, \ldots\\ \gamma_\nu, a_1+s-1, a_2+s, \ldots\end{matrix}\right.$$

$$\left.\begin{matrix}b_q+s, \tfrac{1}{2}(\rho_1+\rho_2), \tfrac{1}{2}(\rho_1+\rho_2-1)\\ a_p+s-2, \rho_1, \rho_2, \rho_1+\rho_2-1\end{matrix}\;\middle|\; x\right),$$

$\lambda+q\leq+\nu+p+2.$

$$(4.3)\qquad \frac{(b_1-a_1-1)\,(b_1+s-1)\,(b_2+s-1)}{(1-a_1-s)\,(a_v+s-1)}\;{}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda,\, b_1+s,\, \ldots\\ \gamma_\nu,\, a_1+s,\, \ldots\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s,\, \tfrac{1}{2}(\rho_1+\rho_2),\, \tfrac{1}{2}(\rho_1+\rho_2-1)\\ a_p+s,\, \rho_1,\, \rho_2,\, \rho_1+\rho_2-1\end{matrix}\,\middle|\, x\right)$$

$$=\frac{(b_1+s-1)\,(b_2+s-1)\,(b_1+s)}{(1-a_1-s)\,(a_p+s-1)}\;{}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda,\, b_1+s+1,\, b_2+s,\, \ldots\\ \gamma_\nu,\, a_1+s,\, a_2+s,\, \ldots\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s,\, \tfrac{1}{2}(\rho_1+\rho_2),\, \tfrac{1}{2}(\rho_1+\rho_2-1)\\ a_p+s,\, \rho_1,\, \rho_2,\, \rho_1+\rho_2-1\end{matrix}\,\middle|\, x\right)$$

$$+\,(b_I+s-\mathrm{VQ}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda,\, b_1+s,\, b_2+s-1,\, b_2+s,\, \ldots\\ \gamma_\nu,\, a_1+s,\, a_2+s,\, \ldots\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s,\, \tfrac{1}{2}(\rho_1+\rho_2),\, \tfrac{1}{2}(\rho_1+\rho_2-1)\\ a_p+s-1,\, \rho_1,\, \rho_2,\, \rho_1+\rho_2-1\end{matrix}\,\middle|\, x\right)$$

$$+\frac{(a_p-b_1)\,(a_p-b_2)\,(a_p-1)}{(a_p+s-1)}\;{}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda,\, b_1+s-1,\, b_2+s-1,\, \ldots\\ \gamma_\nu,\, a_1+s-1,\, a_2+s,\, \ldots\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s,\, \tfrac{1}{2}(\rho_1+\rho_2),\, \tfrac{1}{2}(\rho_1+\rho_2-1)\\ a_p+s,\, \rho_1,\, \rho_2,\, \rho_1+\rho_2-1\end{matrix}\,\middle|\, x\right)$$

$$+\,(b_2-a_p)\;{}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda,\, b_1+s-1,\, b_2+s-1,\, b_3+s,\, \ldots\\ \gamma_\nu,\, a_1+s-1,\, a_2+s,\, \ldots\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s,\, \tfrac{1}{2}(\rho_1+\rho_2),\, \tfrac{1}{2}(\rho_1+\rho_2-1)\\ a_p+s-1,\, \rho_1,\, \rho_2,\, \rho_1+\rho_2-1\end{matrix}\,\middle|\, x\right),$$

$$\lambda+q\leq+\nu+p+2.$$

$$(4.4)\qquad \frac{(b_1-a_1)\,(b_1+s-1)\,(b_2+s-1)}{(1-a_1-s)\,(a_p+s-1)}\;{}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda,\, b_1+s,\, \ldots\\ \gamma_\nu,\, a_1+s,\, \ldots\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s,\, \tfrac{1}{2}(\rho_1+\rho_2),\, \tfrac{1}{2}(\rho_1+\rho_2-1)\\ a_p+s,\, \rho_1,\, \rho_2,\, \rho_1+\rho_2-1\end{matrix}\,\middle|\, x\right)$$

$$=\frac{(b_1+s-1)\,(b_2+s-1)}{(1-a_1-s)}\;{}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda,\, b_1+s,\, \ldots\\ \gamma_\nu,\, a_1+s,\, \ldots\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s,\ \tfrac{1}{2}(\rho_1+\rho_2),\ \tfrac{1}{2}(\rho_1+\rho_2-1) \\ a_p+s-1,\ \rho_1,\ \rho_2,\ \rho_1+\rho_2-1 \end{matrix}\ \right|\ x\Bigg)$$

$$+\frac{(b_1-a_p)\,(b_1-a_1)\,(b_2+s-1)}{(1-a_1-s)\,(a_p+s-1)}\ {}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda,\ b_1+s-1,\ b_2+s,\ \ldots \\ \gamma_\nu,\ a_1+s,\ a_2+s,\ \ldots\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s,\ \tfrac{1}{2}(\rho_1+\rho_2),\ \tfrac{1}{2}(\rho_1+\rho_2-1) \\ a_p+s,\ \rho_1,\ \rho_2,\ \rho_1+\rho_2-1 \end{matrix}\ \right|\ x\Bigg)$$

$$+\,(1+b_2-a_p)\ {}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda,\ b_1+s-1,\ b_2+s-1,\ b_3+s,\ \ldots \\ \gamma_\nu,\ a_1+s-1,\ a_2+s,\ \ldots\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s,\ \tfrac{1}{2}(\rho_1+\rho_2),\ \tfrac{1}{2}(\rho_1+\rho_2-1) \\ a_p+s-1,\ \rho_1,\ \rho_2,\ \rho_1+\rho_2-1 \end{matrix}\ \right|\ x\Bigg)$$

$$+\,(a_p+s-2)\ {}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda,\ b_1+s-1\ \ b_2+s-1,\ \ldots \\ \gamma_\nu,\ a_1+s-1,\ a_2+s,\ \ldots\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s,\ \tfrac{1}{2}(\rho_1+\rho_2),\ \tfrac{1}{2}(\rho_1+\rho_2-1) \\ a_p+s-1,\ \rho_1,\ \rho_2,\ \rho_1+\rho_2-1 \end{matrix}\ \right|\ x\Bigg),$$

$$\lambda+q\leq\nu+p+2.$$

(4.5)
$$\frac{(1+b_1+b_2-a_1-a_p)\,(b_2+s-1)}{(1-a_1-s)\,(a_p+s-1)}\ {}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda,\ b_1+s,\ \ldots \\ \gamma_\nu,\ a_1+s,\ \ldots\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s,\ \dfrac{\rho_1+\rho_2}{2},\ \dfrac{\rho_1+\rho_2-1}{2} \\ a_p+s,\ \rho_1,\ \rho_2,\ \rho_1+\rho_2-1 \end{matrix}\ \right|\ x\Bigg)$$

$$=\frac{(b_1+s+1)\,(b_2+s-1)}{(1-a_1-s)\,(a_p+s-1)}\ {}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda,\ b_1+s+1,\ b_2+s,\ \ldots \\ \gamma_\nu,\ a_1+s,\ a_2+s,\ \ldots\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s,\ \dfrac{\rho_1+\rho_2}{2},\ \dfrac{\rho_1+\rho_2-1}{2} \\ a_p+s,\ \rho_1,\ \rho_2,\ \rho_1+\rho_2-1 \end{matrix}\ \right|\ x\Bigg)$$

$$+\frac{(b_2-a_p)(b_2-a_1)}{(1-a_1-s)(a_p+s-1)}\,{}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda, b_1+s, b_2+s-1, \ldots \\ \gamma_\nu, a_1+s, a_2+s, \ldots\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s, \dfrac{\rho_1+\rho_2}{2}, \dfrac{\rho_1+\rho_2-1}{2} \\ a_p+s, \rho_1, \rho_2, \rho_1+\rho_2-1 \end{matrix}\,\right|\, x\Bigg)$$

$$+\,{}_{\lambda+q+2}F_{\nu+p+3}\left(\begin{matrix}\alpha_\lambda, b_1+s, b_2+s-1, b_3+s, \ldots, \\ \gamma_\nu, a_1+s, a_2+s, \ldots,\end{matrix}\right.$$

$$\left.\begin{matrix} b_q+s, \dfrac{\rho_1+\rho_2}{2}, \dfrac{\rho_1+\rho_2-1}{2} \\ a_p+s-1, \rho_1, \rho_2, \rho_1+\rho_2-1 \end{matrix}\,\right|\, x\Bigg)$$

$\lambda+q\leq\nu+p+2.$