NOTA SOBRE LAS DESIGUALDADES DIFERENCIALES DE ORDEN "N"

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RESUMEN

El objeto de esta nota es añadir algunas diferenciales nuevas de orden n a la vasta literatura de desigualdades diferenciales las cuales puedan ser convenientemente usadas en el análisis de cierta clase de ecuaciones diferenciales de orden n.

1. INTRODUCCION.

Durante los ultimos años se ha publicado un número considerable de trabajos referentes a las desigualdades diferenciales y su aplicación a una variedad de problemas de física (vease [2] - [10] y las referencias en ellas). Se han hecho pocos intentos relativamente para establecer desigualdades diferenciales las cuales puedan aplicarse facilmente a un estudio sistematico de las acuaciones diferenciales de orden n. Sin embargo, algunos desarrollos recientes de la teoría de las ecuaciones diferenciales de orden n se han hecho de urgente interés en el estudio de tales modelos sin necesidad de reducirlos a un sistema de n ecuaciones de primer orden. Nuestro objeto en esta nota es el de establecer algunas desigualdades diferenciales nuevas de orden n que sean aplicables facilmente al estudio del comportamiento del modelo de ecuaciones diferenciales de orden n considerado por Z. Opial [4] sin reducirlo a un sistema de n ecuaciones de primer orden.

2. RESULTADOS PRINCIPALES

En esta sección fijamos nuestros resultados principales en algunas disigualdades diferenciales de orden n fundamentales que pueden usarse como un útil práctico en el análisis de una clase de ecuaciones diferenciales de orden n.

Una desigualdad diferencial de orden n está contenida en el teorema siguiente

TEOREMA 1. Sea $y(t), y'(t), \ldots, y^{(n)}(t)$, y a (t) el valor real de la función continúa no negativa definida como $I = [o, \infty)$ con excepción que $y^{(n-1)}(t)$ es positiva para todos t C I, para la cual se refiera la desigualdad.

$$(1) \qquad y^{(n)}(t) \leq a(t)\, y^{(n-1)}(t)\,[\, y(t) + y'(t) + \ldots + y^{(n-1)}(t)\,],$$

se satisface para toda t C I. si

$$1 - [\, y(o) + y'(o) + \ldots + y^{(n-1)}(o)\,] \int_o^t e^s\, a(s) ds > o,$$

para toda t C I, luego

$$(2) \qquad y^{(n)}(t) \leq a(t)\, Q(t)\, y^{(n-1)}(o) \exp\left(\int_o^t a(s)\; Q(s)\, ds\right),$$

para toda t C I, donde

$$(3) \qquad Q(t) = \frac{e^t\,[\, y(o) + y'(o) + \ldots + y^{(n-1)}(o)\,]}{1 - [\, y(o) + y'(o) + \ldots + y^{(n-1)}(o)\,] \int_o^t e^s\, a(s) ds},$$

para toda t C I.

Demostración. Definida

$$(4) \qquad m(t) = y(t) + y'(t) + \ldots + y^{(n-1)}(t),$$
$$m(o) = y(o) + y'(o) + \ldots + y^{(n-1)}(o),$$

luego diferenciando (4) u usando la propiedad $y^{(n-)}(t) \leq a(t)\; m(t)$. $y^{(n-1)}(t)$ luego de (1) junto con el hecho de $y'(t) + y''(t) + \ldots + y^{(n-1)}(t) \leq m(t)$ y $y^{(n-1)}(t) \leq m(t)$ de (4) vemos que la desigualdad

$$m'(t) \leq m(t) + a(t)\, m^2(t),$$

se satisface para toda t C I, lo cual implica la estimación de m(t) tal que: (vease, [7])

$$(5) \quad m(t) \leq \frac{e^t\,[\, y(o) + y'(o) + \ldots + y^{(n-1)}(o)\,]}{1 - [\, y(o) + y'(o) + \ldots + y^{(n-1)}(o)\,] \int_o^t e^s \cdot a(s) ds} = Q(t),$$

para toda t C I. Ahora substituyendo el valor de m(t) en (1) tenemos

$$(6) \qquad y^{(n)}(t) \leq a(t)\, Q(t)\, y^{(n-1)}(t),$$

para toda t C I. Dividiendo ambos lados de (6) por $y^{(n-1)}(t)$ y luego integrando de o a t obtenemos el valor de $y^{(n-1)}(t)$ tal que

$$(7) \qquad y^{(n-1)}(t) \leq y^{(n-1)}(o) \exp\left(\int_o^t a(s)\, Q(s) ds\right),$$

para toda t C I. El límite deseado en (2) sigue de (1),(5) y (7)

Observamos que usamos la condición $y^{(n-1)}(t) \leq m(t)$ de (4) en lugar de la condición (6) en (1), luego en vista de (5), el resultado obtenido en (2) se reduce a

$$y^{(n)}(t) \leq a(t)\, Q^2(t),$$

para toda t C I.

Se da otra desigualdad diferencial de orden n en el siguiente teorema.

TEOREMA 2. Sea $y(t), y'(t),\ldots, y^{(n)}(t)$, y tal que $a_i(t)$ $(i=o, 1,2,\ldots n)$ sea un valor real no negativo de una función continua definida en I solo que $y^{(n-1)}(t)$ es positivo para toda t C I. para el cual se verifica la desigualdad

$$(8) \quad y^{(n)}(t) \leq \sum_{i=1}^{n} a_i(t)\,(y(t)+y'(t)+\ldots+y^{(n-1)}(t))\,y^{(i-1)}(t)$$

$$+ a_0(t)\,(y(t)+y'(t)+\ldots+y^{(n-1)}(t)),$$

se cumple para toda t C I. Si

$$1- [y(o)+y'(o)+\ldots+y^{(n-1)}(o)]\int_o^t \Big(\sum_{i=1}^{n} a_i(s)\Big)\exp\Big(\int_o^s [1+a_0(t)]\,dt\Big)ds > o$$

para toda t C I. Luego

$$(9) \quad y^{(n)}(t) \leq \Big(\sum_{i=1}^{n} a_i(t)\Big)R^2(t) + a_o(t)R(t),$$

Para toda t C I, Donde

$$(10)\ R(t)=\frac{[y(o)+y'(o)+\ldots+y^{(n-1)}(o)]\exp\big(\int_0^t [1+a_o(s)]\,ds\big)}{1-[y(o)+y'(o)+\ldots+y^{(n-1)}(o)]\int_o^t\big(\sum_{i=1}^{n} a_i(s)\big)\exp\big(\int_o^s[1+a_0(t)]dt\big)ds},$$

para toda t C I,

Demostración. Se define

$$(11) \quad m(t) = y(t) + y'(t)+\ldots+y^{(n-1)}(t),$$

$$m(o) = y(o) + y'(o)+\ldots+y^{(n-1)}(o),$$

Luego diferenciando (11) u usando el caso de

$$y^{(n)}(t) \leq \sum_{i=1}^{n} a_i(t)\,m(t)\,y^{(i-1)}(t) + a_0(t)\,m(t)$$

para (8), junto al caso de

$$y'(t) + y''(t)+\ldots+y^{(n-1)}(t) \leq m(t)$$

y

(12) $y^{(i-1)}(t) \leq m(t), (i=1,2,...,n)$

de (11) vemos que la desigualdad

$$m'(t) \leq [1 + a_o(t)]m(t) + (\sum_{i=1}^{n} a_i(t))m^2(t)$$

se satisface para toda t C I, lo cual implica un valor de m(t) tal que

$$(13)\ m(t) \leq \frac{[y(o)+y'(o)+...+y^{(n-1)}(o)]\exp(\int_0^t [1+a_o(s)]ds)}{1-[y(o)+y'(o)+...+y^{(n-1)}(o)]\int_0^t (\sum_{i=1}^{n} a_i(s))\exp(\int_0^s [1+a_o(t)]dt)ds} = R(t)$$

para toda t C I. El deseado límite en (9) se deduce de (8), (12) y (13)

Luego demostramos la desigualdad diferencial de orden n siguiente que puede ser conveniente para ciertas aplicaciones.

TEOREMA 3. Sea y(t), y'(t),...,$y^{(n)}(t)$, b(t) y cada $a_i(t)$ (i=1,2,...,n) es un valor real no negativo de una función continua definida en I para la cual se satisface la desigualdad.

$$(14)\ y^{(n)}(t) \leq a_1(t)y(t) + a_2(t)y'(t) + ... + a_n(t)y^{(n-1)}(t) + b(t)(y(t) + y'(t) + ... + y^{(n-1)}(t)),$$

se satisface para toda t C I. Luego

$$(15)\quad y^{(n)}(t) \leq [b(t) + \sum_{i=1}^{n} a_i(t)][y(o)+y'(o)+...+y^{(n-1)}(o)] .\exp(\int_0^t [1 + b(s) + \sum_{i=1}^{n} a_i(s)]ds),$$

para toda t C I.

La demostración de este teorema sigue las mismas ideas que la demostración del teorema 2, con modificaciones apropiadas, y dejamos los detalles al lector

3. **Una aplicación.**-En esta parte indicamos una aplicación simple de

nuestro teorema 2 para obtener el límite de la derivada n de la solución de una clase de ecuaciones diferenciales de orden n consideradas por Z Opial [4] de la forma (vease, [1])

$$(16)\quad y^{(n)}(t) = \sum_{i=1}^{n} f_i(t,y(t),y'(t),\ldots,y^{(n-1)}(t))\, y^{(i-1)}(t)$$
$$+ k(t,y(t), y'(t),\ldots,y^{(n-1)}(t)),$$
$$y^{(p)}(o) = y_o^p \;, o \leq p \leq n-1,$$

Donde $y(t), y'(t),\ldots,y^{(n)}(t)$, f_i $(i=1,2,\ldots,n)$ and k son los elementos de R^n, y del espacio Euclideo de n dimensiones, continuas en sus respectivos dominios de definiciones e y_o es una constante positiva dada.

Sea /./ una designacion conveniente de norma de R^n . Supongamos que las funciones. f_i $(i=1,2,\ldots,n)$ y k en (16) satisfacen.

$$(17)\quad |f_i(t,y(t),y'(t),\ldots,y^{(n-1)}(t)|$$
$$\leq a_i(t)[\,|y(t)| + |y'(t)| + \ldots + |y^{(n-1)}(t)|\,],$$

$$(18)\quad |k(t,y(t),y'(t),\ldots,y^{(n-1)}(t))|$$
$$\leq a_o(t)[\,|y(t)| + |y'(t)| + \ldots + |y^{(n-1)}(t)|\,]$$

Para toda T ϵ I, donde $a_i(t)$ $(i=o,1,2,\ldots,n)$ son funciones continuas no negativas definidas en I. Isando (17) y (18) en (16) y aplicando el teorema 2 tenemos

$$(19)\quad |y^{(n)}(t)| \leq \Big(\sum_{i=1}^{n} a_i(t)\Big) Z^2(t) + a_o(t)\, Z(t),$$

para todo t C I, donde

$$(20)\quad Z(t) = \frac{[\,1+y_o+y_o^2+\ldots+y_o^{n-1}]\exp\left(\int_o^t [\,1+a_o(s)\,]\,ds\right)}{1-[\,1+y_o+y_o^2+\ldots+y_o^{n-1}]\int_o^t \Big(\sum_{i=1}^{n} a_i(s)\Big)\exp\left(\int_o^s [\,1+a_o(t)\,]\,dt\right)ds};$$

$$(21)\quad 1-\left[1+y_0+y_0^2+y_0^{n-1}\right]\int_0^t\left(\sum_{i=1}^{n}a_i(s)\right)\exp\left(\int_0^s\left[1+a_0(t)\right]dt\right)ds>1,$$

para toda t C I. Luego integrado (19) de O a t, n veces, se puede establecer facilmente el límite de la solución de $y^{(t)}$ de (16).

Finalmente observamos la utilidad de las disigualdades establecidas en los teoremas 1-3; se hace aparente si consideramos y (o), y'(o),..., $y^{(n-1)}$ (o), a_i (t) (i=0,1,2,...,n) y b(t) son conocidas y y(t), y'(t),...,$y^{(n)}$ (t) son desconocidas, por ejemplo las desigualdades establecidas en los teoremas 1-3 nos dan los límites en terminos de las funciones conocidas que hacen mayor $y^{(n)}$ (t) y por consecuencia y(t) después de n integraciones. En trabajos futuros ilustraremos otras aplicaciones de nuestras desigualdades a otros problemas de la teoría de las ecuaciones diferenciales de orden n.

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A NOTE ON n th ORDER DIFFERENTIAL INEQUALITIES

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Summary. The object of this note is to add some new n th order diferential inequalities to the vast literature on differential inequialities which can be convenientrly used in the analysis of a class of n the order differential equations.

1. **Intrudiction.** During the last several years substantial number of papers have been published dealing with differential inequalities and their applications to a variety of mathematical problems of physical interest (see, [2] - [10] and the references therein). Relatively, few attempts have been made to establish n th order differential enequialities which are easily applicable to a systematic study of n the order differential equiations. However, some recent developments in the theory of n th order differential equiations have become of compelling interest in the study of such models without reducing them to a system of n equations of the first order. Our aim in this note is to establish some new n th order differential inequalities which are easily applicable to study the behavior of n th order differential equiation model considered by Z. Opial in [4] , without reducing it to a system of n equations of the first order.

2. **Main Results.** In this section we establish our main results on some fundamental n th order differential inequialities which can be used as handy tools in the analysis of a class of n th order differential equiations.

A useful n th order differential inequality is embodied in the following theorem.

THEOREM 1. Let $y(t), y'(t), \ldots, y^{(n)}(t)$, **and** $a(t)$ **be realvalued nonnegative continuous functions defined onf** $I \doteq [0, \infty)$ **except that** $y^{(n-1)}(t)$ **is positive for all** $t \in I$, **for which the inequality**

$$(1) \qquad y^{(n)}(t) \leq a(t) y^{(n-1)}(t) [y(t) + y'(t) + \ldots + y^{(n-1)}(t)],$$

holds for all t $\in$ I. If

$$1- [y(o)+y'(o)+\dots+y^{(n-1)}(o)] \int_o^t e^s \; a(s)ds > o,$$

for all $\in$ I, where

$$(2) \quad y^{(n)}(t) \leq a(t)\, Q(t)\, y^{(n-1)}(o) \exp\left(\int_o^t a(s) \; Q(s)\, ds\right),$$

for all t $\in$ I. where

$$(3) \quad Q(t) = \frac{e^t [y(o)+y'(o)+\dots+y^{(n-1)}(o)]}{1- [y(o)+y'(o)+\dots+y^{(n-1)}(o)] \int_o^t e^s \; a(s)ds},$$

for all t $\in$ I.

Proof. Define

$$(4) \quad m(t) = y(t) + y'(t) + \dots + y^{(n-1)}(t),$$
$$m(o) = y(o) + y'(o) + \dots + y^{(n-1)}(o),$$

then differentiating (4) and using the fact that $y^{(n)}(t) \leq a(t)\, m(t)$. $y^{(n-1)}(t)$ from (1) together with the facts that $y'(t) + y''(t) + \dots + y^{(n-1)}(t) \leq m(t)$ and $y^{(n-1)}(t) \leq m(t)$ from (4) we see that the inequality

$$m'(t) \leq m(t) + a(t)\, m^2(t),$$

is satisfied for all t $\in$ I, which implies the estimation for m (t) suche that (see, [7])

$$(5) \quad m(t) \leq \frac{e^t [y(o) + y'(o) + \dots + y^{(n-1)}(o)]}{1- [y(o) + y'(o) + \dots + y^{(n-1)}(o)] \int_o^t e^s \; a(s)ds} = Q(t),$$

for all $t \in I$. Now, substituting this value of m(t) in (1) we have

$$(6) \qquad y^{(n)}(t) \leq a(t)Q(t)y^{(n-1)}(t),$$

for all $t\in I$. Dividing both sides of (6) by $y^{(n-1)}(t)$ and then integrating from o to t we obtain the estimate ofr $y^{(n-1)}(t)$ such that

$$(7) \qquad y^{(n-1)}(t) \leq y^{(n-1)}(o)\exp\left(\int_o^t a(s)Q(s)ds\right),$$

for all $t \in I$. The desired bound in (2) follows from (1), (5), and (7).

We note that, if we use the condition $y^{(n-1)}(t) \leq m(t)$ from (4) in place of condition (6) in (1), then in view of (5), the bound obtained in (2) reduces to

$$y^{(n)}(t) \leq a(t)Q^2(t),$$

for all $t \in I$.

Another interesting and useful n th order differential inequality is given in the following theorem.

THEOREM 2. Let $y(t), y'(t), \ldots, y^{(n)}(t)$, **and eache** $a_i(t)$ $(i = o,1,2,..n)$ **be real-valued nonnegative continuous functions defined on I expect that** $y^{(n-1)}(t)$ **is positive ofr all** $t \in I$, **for which the inequality**

$$(8) \quad y^{(n)}(t) \leq \sum_{i=1}^{n} a_i(t)\,(y(t)+y'(t)+\ldots+y^{(n-1)}(t))\,y^{(i-1)}(t)$$
$$+\, a_o(t)(y(t)+y'(t)+\ldots+y^{(n-1)}(t)),$$

holds for all $t \in I$. **If**

$$1\text{-} \quad [y(o)+y'(o)+\ldots+y^{(n-1)}(o)]\int_o^t \left(\sum_{i=1}^{n} a_i(s)\right)\exp\left(\int_o^s [1+a_o(t)]dt\right)ds > o$$

for all $t \in I$, **then**

$$(9) \qquad y^{(n)}(t) \leq \left(\sum_{i=1}^{n} a_i(t)\right)R^2(t) + a_o(t)R(t),$$

for all $t \in I$, where

$$(10)\ R(t) = \frac{[\,y(o)+y'(o)+\ldots+y^{(n-1)}(o)\,]\exp\left(\int_0^t [\,1+a_0(s)\,]ds\right)}{1-[\,y(o)+y'(o)+\ldots+y^{(n-1)}(o)\,]\int_0^t \left(\sum_{i=1}^{n} a_i(s)\right)\exp\left(\int_0^s [\,1+a_0(t)]dt\right)ds},$$

for all $t \in I$.

Proof. Define

$$(11)\quad m(t) = y(t) + y'(t)+\ldots+y^{(n-1)}(t),$$
$$m(o) = y(o) + y'(o)+\ldots+y^{(n-1)}(o),$$

then differentiating (11) and using the fact that

$$y^{(n)}(t) \leq \sum_{i=1}^{n} a_i(t)\,m(t)\,y^{(i-1)}(t) + a_0(t)\,m(t)$$

from (8) together with the facts that

$$y'(t) + y''(t)+\ldots+y^{(n-1)}(t) \leq m(t)$$

and

$$(12)\quad y^{(i-1)}(t) \leq m(t),\ (i=1,2,\ldots,n)$$

from (11) we see that the inequality

$$m'(t) \leq [\,1 + a_0(t)\,]\,m(t) + \left(\sum_{i=1}^{n} a_i(t)\right) m^2(t)$$

is satisfied for all $t \in I$, which implies the estimation for m(t) souch that

$$(13)\ m(t) \leq \frac{[\,y(o)+ y'(o)+\ldots+y^{(n-1)}(o)\,]\exp\left(\int_0^t [\,1+a_0(s)\,]ds\right)}{1-[\,y(o)+y'(o)+\ldots+y^{(n-1)}(o)]\int_0^t \left(\sum_{i=1}^{n} a_i(s)\right)\exp\left(\int_0^s [\,1+a_0(t)\,]dt\right)ds} = R(t)$$

for all $t \in I$. The desired bound in (9) follows from (8), (12) and (13).

We next estalish the following n th order differential inequality which may be convenient in some applications.

THEOREM 3. **Let** $y(t)$, $y'(t), \ldots, y^{(n)}(t)$, $b(t)$, **and each** $a_i(t)$ $(i=1,2,\ldots,n)$ **be real-valued nonnegative continuous funcitions defined on** i, **for which the inequality**

$$(14)\quad y^{(n)}(t) \leq a_1(t)y(t) + a_2(t)y'(t) + \ldots + a_n(t)\, y^{(n-1)}(t)$$
$$+ b(t)\,(y(t) + y'(t) + \ldots + y^{(n-1)}(t)),$$

Holds for all $t \in I$. **Then**

$$(15)\quad y^{(n)}(t) \leq [\, b(t) + \sum_{i=1}^{n} a_i(t)\,][\, y(o) + y'(o) + \ldots + y^{(n-1)}(o)\,]$$
$$.\exp\left(\int_o^t [\,1 + b(s) + \sum_{i=1}^{n} a_i(s)\,]\,ds\right),$$

for all $t \in I$.

The proof of this theorem follows by the similar argument as in the proof of Theorem 2 with suitable modifications, and we leave the datails to the reader.

3. **An Application.** In thes section we indicate a simple application of our Theorem 2 to obtain the bound on n th derivative of the solution of a class of n th order differential equation considered by Z. Opial [4] of the form (see, [1])

$$(16)\quad y^{(n)}(t) = \sum_{i=1}^{n} f_i(t, y(t), y'(t), \ldots, y^{(n-1)}(t))\, y^{(i-1)}(t)$$
$$+ k(t, y(t), y'(t), \ldots, y^{(n-1)}(t)),$$
$$y^{(p)}(o) = y_o^p, \quad o \leq p \leq n-1,$$

where $y(t)$, $y'(t), \ldots, y^{(n)}(t)$, f_i $(i=1,2,\ldots,n)$ and k are the elments of R^n, an n-dimensional Euclidean space, and continuous on the respective domains of their definitions and y_o is a given positive constant.

Let /./ denote some convenient norm on R^n. Suppose that the functions f_i $(i=1,2,\ldots,n)$ and k in (16) satisfy

(17) $$|f_i(t,y(t),y'(t),\ldots,y^{(n-1)}(t)|$$

$$\leq a_i(t)[\,|y(t)|+|y'(t)|+\ldots+|y^{(n-1)}(t)|\,],$$

(18) $$|k(t,y(t),y'(t),\ldots,y^{(n-1)}(t))|$$

$$\leq a_0(t)[\,|y(t)|+|y'(t)|+\ldots+|y^{(n-1)}(t)|\,]$$

for all $t \in I$, where $a_i(t)$ $(i=o,\ 1,\ 2,\ldots n)$ are nonnegative continuous functions defined on I. Using (17) and (18) in (16) and applying Theorem 2 we have

(19) $$|y^{(n)}(t)| \leq (\sum_{i=1}^{n} a_i(t))Z^2(t) + a_0(t)Z(t),$$

for all $t \in I$, where

(20) $$Z(t) = \frac{[1+y_0+y_0^2+\ldots+y_0^{n-1}]\exp(\int_0^t[1+a_0(s)]ds)}{1-[1+y_0+y_0^2+\ldots+y_0^{n-1}]\int_0^t(\sum_{i=1}^{n}a_i(s))\exp(\int_0^s[1+a_0(t)]dt)ds};$$

(21) $$1-[1+y_0+y_0^2+y_0^{n-1}]\int_0^t(\sum_{i=1}^{n}a_i(s))\exp(\int_0^s[1+a_0(t)]dt)ds > 1,$$

for all $t \in I$. Further integrating (19) from o to t, n times one can very easily establish the bund on the solution $y(t)$ of (16).

Finally, we note tha the usefulness of the differential inequalities esblished in Theorems 1-3 becomes apparent if we consider

$y(o), y'(o), \ldots, y^{(n-1)}(o)$, $a_i(t)$ $(i=o,1,2,\ldots,n)$ and $b(t)$ are known and $y(t)$, $y'(t),\ldots,y^{(n)}(t)$ are unknown functiones, i.e. the inequalities established in Theorems 1-3 gives us the bounds in terms of the known functions which majoriges $y^{(n)}(t)$ and consequently $y(t)$ after n times integration. In our future papers we wish to illustrate further applications of our inequalities to some other problems in the theory of n th order differential equations.

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